ANALYSIS AND EXPERIMENTAL VERIFICATION OF POLYMER MELT NOZZLE PRESSURE DROP IN FUSED FILAMENT FABRICATION ADDITIVE MANUFACTURING

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Abstract

Quantifying nozzle pressure drop in Fused Filament Fabrication (FFF) polymer melt flow can be used to understand the effect of nozzle geometry and polymer melt rheology on the deposition process. Existing FFF nozzle pressure drop models focus on various sections within the nozzle which includes both conical and cylindrical flow geometries having regions of shear and extensional flow. Unfortunately, little attention has been given to the validation of various FFF nozzle flow models for predicting the pressure drop through the nozzle during material deposition. Our study develops a custom filament force measuring device that is used to obtain the pressure drop through commercially available FFF nozzles. The measured pressure drop is then compared to compute results obtained using FFF nozzle flow models that incorporate various polymer melt rheology models including the Power Law, Cross Law, Bird-Carreau, and Carreau-Yasuda models. We also examine a model that includes an extensional flow component. Our computed results show that the FFF nozzle has the highest pressure drop in the conical contraction and capillary regions, and that polymer melt flow in the inlet cylinder cannot be ignored. These computations are then used to assess the relative accuracy of various FFF nozzle melt flow models.

Introduction

In the Fused Filament Fabrication process, polymer feedstock in filament form is delivered via the pinch roller mechanism into a liquefier then is melted and extruded though the nozzle. The significance of the studies on correlating the rheology of polymers to a certain AM technique, i.e., FFF, has been discussed in [1]. However, directly using the rheological data obtained from the standard testing to represent the rheology of the polymeric materials processed in FFF is unspecific. This is because the measuring geometry of either rotational rheometer or capillary rheometer is carefully designed with the purpose of characterizing the materials such that the material under testing can be recognized in steady state and is fully developed. However, the inside geometry of a typical FFF nozzle is more complicated (same applied to the polymer melts flow inside) and usually has very high tolerance of manufacturing. Taking advantage of the shear-thinning behavior of polymeric feedstock the FFF nozzle is usually machined with a conical-cylindrical die as the transition region to increase the shear rate of the molten material flow from the upper cylindrical die into the capillary die at the nozzle tip. Therefore, the real-time rheology evaluation is critical to inspect the rheology of the filaments specifically in FFF.

The processing mechanism of FFF is analogous to the capillary rheometer where the solid portion of the filament acts as a piston to push the molten portion flow through the nozzle. Therefore, measuring the pressure drop is an approach in order to obtain the rheology of materials from an FFF-based device. However, it is impractical to directly measure the pressure drop of motel polymers by applying the pressure or force transducer inside of a small-scale FFF nozzle without interfering the melts flow [2,3]. Our study developed an indirect method to measure the extrusion force of filaments to obtain the pressure drop without interfering the flow inside of nozzle.
The analytical models of pressure drop of polymer melts flow in an FFF nozzle existing in the literature were used to compute pressure drop which is then compared to our experimental data. To date, the analytical models of pressure drop for FFF existing in literatures can be grouped as the Power Law-based pure-shear model developed by Bellini [4] and the model developed by Phan, et al. [5] where the elongation and entrance flow were included. In addition to the Power Law-based pressure drop model, i.e., Bellini model and Phan model. We also present enhanced Generalized Newtonian Fluid (GNF) models using Bird-Carreau and Cross model derived from pressure driven flow in a tube model by Sochi [6]. We also modified the Sochi model to include a Carreau-Yadusa formulation to compute the pressure drop of the FFF nozzle flow.

**Pressure Drop Models**

The flow in an FFF nozzle is often viewed as a pressure driven flow. Hence the pressure, especially the pressure drop through the nozzle, of such flow is of importance for describing and understanding the polymer melt flow behavior. As a result, efforts have been made to develop the analytical models for pressure drop as a function of volumetric flow rate or the apparent shear rate within an FFF nozzle. Only a few of the models have been proposed that consider the polymer melt flow in an FFF nozzle [4,5], where the main focus is on the conic-cylindrical region and the capillary region [7,8,9]. This section provides the description of the existing FFF pressure drop models and includes the proposed enhanced GNF models based on work by Sochi.

A cross-section of the FFF nozzle used in this study appears in Figure 1(a). The inside flow cavity geometry consists of the upper tube, the conical region, and a short capillary die. We assume that the overall pressure drop can be simplified as the sum of the pressure drops within each region (i.e., $\Delta P_I$, $\Delta P_{II}$, $\Delta P_{III}$) as outlined in Figure 1(b).

![Figure 1](image)

*Figure 1: (a) Inside geometry of the FFF nozzle used in this study; (b) Schematic of a typical FFF nozzle; (c), Approximation of the converging flow by splitting the flow domain into infinite small cylinders.*

**Bellini Model**

Bellini [4] proposed a 3-region model as shown in in Figure 1(b) assuming the polymer melt flow could be modeled as a purely viscous Power Law fluid. In Bellini’s approach, the pressure drop through an FFF nozzle flow was evaluated from the shear stress on the inside wall of the nozzle. To adopt the Bellini model the necessary assumptions were made as follows,

1. Incompressible polymer and polymer composite melt flow;
2. Polymer melts exhibit a laminar pure shear flow;
3. No-slip boundary condition at the nozzle wall;
4. Isothermal fluid flow condition;
5. Gravity of extrudate is negligible;
6. Polymer melt flow is in steady state in each region;
7. The pressure at the nozzle outlet is equal to zero;
8. Flow entrance and exit effects are negligible;
9. The Power Law model is applicable to polymer melt flow in the nozzle during extrusion;
10. The Reynolds number of the flow is small making it possible to ignore inertia effect;
11. The flow is axisymmetric with respect to the z-axis.

where the isothermal assumption is supported by the efficient temperature control in which the temperature fluctuation during extrusion is within 0.3°C of our device such that the viscosity can be regarded as the shear rate dependent only. The Reynolds number for each material was calculated which has the magnitude of $10^{-7} \sim 10^{-6}$. These assumptions were also applied to develop the enhanced GNF models.

To address the pressure drop in the coni-cylinder (i.e., region II), Bellini assumed the flow domain in this region are the superposition of infinite small tube flow domains. The resulting analytical equation of pressure drop in each region in Figure 1(b) are given as follows,

\[ \Delta P_I = 2kL_1 \left( \frac{Q(1/n+3)}{nR_1^{3/n}} \right)^n, \]  
\[ \Delta P_{II} = \frac{2k\cot\frac{\beta}{2}}{3n} \left( \frac{Q(1/n+3)}{\pi} \right)^n (R_2^{3n} - R_1^{3n}), \] 
\[ \Delta P_{III} = 2kL_3 \left( \frac{Q(1/n+3)}{nR_2^{3/n+1}} \right)^n \]

where \( n \) and \( k \) are Power Law parameters, \( R_1, R_2, L_1, L_3, \) and \( \beta \) are the dimensional parameters of the nozzle which was outlined in Figure 1(b), \( Q \) is the volumetric flow rate, \( \Delta P \) is the overall pressure drop of molten polymer in the nozzle. The individual pressure drops in Equations 1 through 3 are summed to obtain the overall pressure drop in the FFF nozzle as

\[ \Delta P = \Delta P_I + \Delta P_{II} + \Delta P_{III} = 2k \left( \frac{Q(1/n+3)}{\pi} \right)^n \left[ \frac{L_1}{R_1^{3/n+2}} + \frac{\cot\frac{\beta}{2}}{3n} (R_2^{3n} - R_1^{3n}) + \frac{L_3}{R_2^{3/n+1}} \right] \]  

**Phan Model**

More recently, Phan, et al. [5] proposed an FFF nozzle pressure drop model that considered an extensional flow contribution in the conical section of the nozzle. Phan employed results by Cogswell who developed an analytical model for predicting the pressure drop of converging flow in polymer processing, which has been suggested to be capable of predicting the pressure drop within ±20% [7]. Phan, et al. [5] derived a hybrid model by modifying the Cogswell model [7] for predicting pressure drop of flow in conical die but using Boles model [8] for abrupt entrance effect from the conical reservoir into the orifice. In addition to applying the Power Law to the simple shear flow, they assumed this relation also suitable to the elongation flow, that is

\[ \eta_e = H \dot{\varepsilon}^{m-1} \]

where \( \eta_e \) is the extension viscosity, \( H \) and \( m \) are the parameters of Power Law for extension flow here, \( \dot{\varepsilon} \) is the strain rate of flow under extension. Their experimental work found that the Trouton ratio \( Tr = H/k \) required a value of 5 for PLA. Values of the Trouton ratio was set to 3 in the Cogswell model. Also, Phan, et al. assumed \( m = n \) and used a generalized Bernoulli equation to obtain the pressure loss in the nozzle as a combination of simple shear.
\[ \Delta P_{II} = \frac{2k}{3n \tan\left(\frac{\pi}{2}\right)} \left(1 - \left(\frac{R_z}{R_i}\right)^{3n}\right) \dot{Y}_a^n \]  

(6)

simple elongation

\[ \Delta P_{IIe} = H \frac{2^{1-m} \tan\left(\frac{\beta}{2}\right)^m}{3m} \left(1 - \left(\frac{R_z}{R_i}\right)^{3m}\right) \dot{Y}_a^m \]  

(7)

and entrance loss

\[ \Delta P_o = \frac{1.18k}{n^{0.7}} \left[ \frac{Q(3n + 1)}{\pi n R_z^3} \right]^n \]  

(8)

where \( \dot{Y}_a \) is the apparent shear rate, \( \dot{Y}_a = 4Q/\pi R_z^3 \), and \( \Delta P_o \) is an empirical equation used to describe the entrance effect suggested by Boles [6]. Hence, the overall pressure drop in the conical region of the Phan model is written as

\[ \Delta P_{II} = \Delta P_{II} + \Delta P_{IIe} + \Delta P_o \]  

(9)

Additionally, they include the pressure drop contribution \( \Delta P_{III} \) in the capillary region which is the same expression in Equation (3), however, the contribution of pressure drop in region I was ignored.

Enhanced GNF Model

Even though the Power Law model is sufficient to describe the shear thinning behavior of polymer melts processed at high shear rate, it is important to consider other GNF models which are commonly employed to describe the purely viscous flow of polymer melt. To this end, we consider work by Sochi [6] who derived analytical expressions for pressure drop of polymer melt flow in a circular pipe for Bird-Carreau and Cross fluids model. Applying the same approach, we extended Sochi’s work to obtain expressions for pressure drop through a circular tube for the more general Carreau-Yasuda model. The Bird-Carreau, Cross, and Carreau-Yasuda viscosity models are given respectively as

\[ \eta(\dot{\gamma}) = \eta_\infty + (\eta_0 - \eta_\infty)(1 + \lambda^2 \dot{\gamma}^2)^{-\frac{n-1}{2}}, \]  

(10)

\[ \eta(\dot{\gamma}) = \eta_\infty + \frac{(\eta_0 - \eta_\infty)}{(1 + \dot{\gamma}^n)^n} \]  

(11)

and

\[ \eta(\dot{\gamma}) = \eta_\infty + (\eta_0 - \eta_\infty)(1 + \lambda^a \dot{\gamma}^a)^{-\frac{n-1}{a}} \]  

(12)

where \( \eta_\infty \) is the infinite viscosity, \( \lambda \) is the Power Law index, \( a \) is an exponential parameter used to determine the transition from Newtonian plateau to shear thinning region. It has to be mentioned that we assume \( \eta_\infty = 0 \) as is common for polymer melts. Sochi applied the WRMS method [8] where the volumetric flow rate of GNF is given as

\[ \frac{Q_r^3}{\pi R^3} = \int_0^\tau_0 \dot{\gamma} \tau^2 d\tau \]  

(13)

where \( \tau_w \) is the wall shear stress, \( \tau \) is the shear stress, and \( R \) is the radius of the tube. Substituting Equation (5-7) into the constitutive equation \( \tau = \eta(\dot{\gamma})\dot{\gamma} \), and following mathematical derivation, the integral in Equation (13) may be used to obtain
\[ I_{\text{Bird-Carreau}} = \eta_0^2 \left( -2 + \frac{(\gamma_w \lambda^2)^2}{(1 + (\gamma_w \lambda)^n)^2} \left( 2 - 3 (1 + n) \gamma_w^2 \lambda^2 + 3n (1 + 3n) \gamma_w^4 \lambda^4 \right) \right), \] (14)

\[ I_{\text{Cross}} = -\frac{\gamma_w^4}{12} \eta_0^2 \left( 4 \frac{\gamma_w \lambda}{(1 + (\gamma_w \lambda)^n)^3} + \frac{2}{n(1 + (\gamma_w \lambda)^n)^2} + \frac{4(-2 + n)}{n^2(1 + (\gamma_w \lambda)^n)^2} + \frac{(8 - 6n + n^2) \gamma_w^2 \lambda^2}{n^2} \right), \] (15)

and

\[ I_{\text{Carreau-Yaduda}} = \frac{\gamma_w^4}{4} \eta_0^2 \left( \frac{3}{\gamma_w \lambda} \frac{\Gamma(\frac{3 + a - 3n}{a}, \frac{4 + a}{a}, - \gamma_w \lambda a)}{\Gamma(\frac{3 + a - 3n}{a}, \frac{4 + a}{a}, - \gamma_w \lambda a)} + \frac{4n \gamma_w \lambda^2 \gamma_w^4 \lambda^4}{4a} \right). \] (16)

where \( I = \int_0^L \gamma \tau^2 dz, \) is the hypergeometric function where its real part can be calculated via MATLAB (MathWorks, Inc., Natick, MA) hypergeom function, \( \gamma_w \) is the wall shear rate. In the end, the pressure drop of polymer melts in a tube can be expressed as the function of volumetric flow rate for any of these GNF models as

\[ \Delta P = \left( \frac{8\pi l^3 f}{Q} \right)^{1/3} \] (17)

Once the geometry of the tube and the pressure drop \( \Delta P \) are known, \( \gamma_w \) can be solved using the desired constitutive equation which yields the following for laminar tube flow

\[ \tau_w = \frac{\Delta P}{2L} = \eta \left( \gamma_w \lambda \right) \gamma_w \] (18)

which requires the roots of this nonlinear equation be obtained. We use \texttt{fzero} in MATLAB to obtain the roots, or when numerical issues arise, we instead use MATLAB optimization solver \texttt{fmincon} was employed to approximate the roots.

To calculate pressure drop using the Sochi-based tube flow model, we adopt Equation (17) to predict \( \Delta P_I \) and \( \Delta P_{II} \) (cf. Figure 1(a, b)). For \( \Delta P_{II} \), we use an approach that is similar to that given in Bellini [4] which is to integrate Equation (17) and (18) simultaneously over \( L_2 \). The shear rate cannot be expressed as the close form such as that given in Equation (2), instead, we numerically calculate the pressure drop over the conical section as

\[ \Delta P_{II} = \int_{L_2}^{L_2} \Delta P \left( R(z), \gamma_w(R(z)) \right) dz \] (19)

By replacing the integral term (i.e. \( I \)) in Equation (14-16) we obtained the modified Sochi models of the FFF nozzle flow corresponding to different GNF models.

**Materials and Experiments**

Four neat polymers which include two brands of ABS (3DXTECH and Triptech Plastic), PLA (3DXTECH), and Amphora (Triptech Plastic) in filaments having a diameter of 1.75 mm were used in this study. Rheological property measurements for all materials were performed using the HAAKE MARS 40 (Thermo Fisher Scientific, Waltham, MA) cone and plate rheometer. The extreme frequency window set as 0.1 – 100 Hz for two brands of ABS and 0.5-100 Hz for PLA and Amphora to guarantee the measurements in the linear viscoelastic region (LVER). The oscillation frequency sweep measurements were repeatedly conducted five times for each material. Typical values for dynamic viscosity as a function of oscillation frequency appear in the plot in Figure 2.
Figure 2: Viscosity curves of four neat polymers measured using MARS 40 rotational rheometer.

The device we developed in this study to measure nozzle flow pressure loss directly uses the commercial and ready-made components of desktop 3D printers. By separating the liquefier and the stepper motor of a direct extruder system, the overall force acting on the nozzle can be measured via a load cell as shown in Figure 3. Assuming the pressure at the nozzle exit is zero, then the resultant force revealed by load cell may be written in terms of the pressure drop over the entire nozzle as

\[ F_f = \Delta P A \quad (20) \]

where \( A \) is the area of nozzle inlet, \( F_f \) is the resultant force.

Figure 3: Measuring protocol of our device
The effective temperature control is needed to make consistent melt flow measurements. This requires an accurate measure of temperature for thermal feedback control. To this end, we use an NTC thermistor and a K-type thermocouple to monitor the temperature simultaneously where the heater block is redesigned to ensure the two temperature sensors are equal-distanced from the nozzle screw. Generally, the built-in framework of a commercial desktop 3D printer monitors the temperature by interpolating the pre-defined temperature-resistance table for a certain thermistor. However, we observed the non-negligible deviation of temperature reading for these thermistors. Hence, we calibrated and the thermistor using the ETC-400A (AMETEK Inc, Berwyn, PA), then verified its accuracy using a K-type thermocouple. We also found that the thermocouple has about 5.5 seconds delay on reading temperature comparing to the thermistor. Considering the small difference of temperature readings from thermistor and thermocouple along with the enormous difference of their resolution (i.e., 0.01°C of thermistor and 0.25°C of thermocouple), thermistor was used to as the temperature sensor reading for our PID controller.

In order to maintain the sufficient and constant heat flux from the nozzle heater to adapt to different filament feeding velocity, the PID control parameters for each material under different feeding rate were calibrated. Then we performed the temperature tests for same material under different delivery speed, and the results show that the maximum temperature fluctuation of the device in operation is about 0.3°C. An example temperature reading appears in Figure 4. Considering the temperature effect on viscosity in Equation (21), we applied the Arrhenius law (Equation (22)) which is valid for temperatures well-above the glass transition to determine the impact from the temperature fluctuation. In these equations, $T_0$ is the reference temperature, (which is also the setpoint of operation temperature in this case), $T$ is the real-time temperature, $\dot{\gamma}$ is the shear rate, $\eta_0$ is the viscosity at $T_0$, $E_0$ is the activation energy, and $R_g$ is the gas constant. According to the published data of activation energy in [10] for PLA and [11] for ABS, it can be inferred that the activation energy of materials assessed in this paper in FFF with the magnitude of $10^2$ KJ/mol. Therefore, the very small fluctuation of temperature yields values of $H(T)$ to be equal to 1 which supports the isothermal assumption we made in applying the math model to predict the rheology of materials in later section.

$$\eta(T, \dot{\gamma}) = H(T)\eta_0(\dot{\gamma})$$  \hspace{1cm} (21)

$$H(T) = \exp\left[\frac{E_0}{R_g\left(\frac{1}{T} - \frac{1}{T_0}\right)}\right]$$ \hspace{1cm} (22)

![Graph](image.png)

Figure 1: Examples of performance of PID control for 3DXTECH ABS under extrusion at 1.5 RPM 230°C.

Before performing the force measurement, we selected and inspected the rollers couple and tension of the spring which is used to add the compression force on filament as it passes through the extruder. The compressive force applied by the teeth of the roller must avoid bucking and
stripping of filament. Also, the stepper motor was adjusted working at precise RPM under 1/32 micro-steps but still capable of providing sufficient torque. We considered that the filament melt flow in the nozzle can be regarded as near steady state with respect to well-controlled temperature as well as the smooth and precise filament feeding process. Unfortunately, the load signal appearing in Figure 5(a) exhibits a level of variation that is too high for reliable force measurement. Therefore, the load cell output signal is analyzed here to gain a better understanding of fluctuations in the signal.

The one-sided frequency spectrum of the force signal is given in Figure 5(b) by performing the Fast Fourier Transform (FFT) where the first dominant frequency component is removed to gain a clear view of other signals. The magnitude of the signal at each frequency was normalized to the scale of one to make it easier to compare the intensity of each frequency component. Two explicit factors causing the fluctuation of the force signal were detected by comparing the data in both time and frequency domain. The frequency bin at 0.01706 Hz was found to be near the rotational frequency of the stepper motor that is 0.0167 Hz at 1 RPM which also matches to the frequency of the peaks in Figure 5(a). This infers that either the idler pulley or the drive gear is not perfectly round, resulting in the insertion velocity of the filament changing over time. Also, the 0.593 Hz bin was found to be equivalent to the frequency of drive gear tooth impact on the filament (36-tooth spinning with speed of 1 RPM ) which is 0.6 Hz . Other higher but non-dominant frequency content may be caused by backflow of the molten material as discussed in Gilmer, et al.’s work [12] or the electrical noise from the circuit were filtered by implementing a low-pass filter in Arduino. The repeatability of the force measurements was demonstrated by averaging the signal over time where the time window was selected which provided a stable temperature reading.

Having the consistent force readings, the experiments on our device were performed for each material with changing filament feeding speed that is from 0.5 RPM to 2.5 RPM with 0.5 RPM as the increment. At each speed, both pressure drop and volumetric flow rate were obtained and further analyzed. The measured relationship between pressure drop and volumetric flow rate appears in Figure 6.
Results and Analysis

Parameters that define each of the GNF models were computed through a curve fitting process using viscosity data (cf. Figure 2) obtained from MARS 40. The curve fitting was performed using Polyflow (ANSYS, Inc., Canonsburg, PA, USA) where the Cox-Merz rule [13] was applied to convert the dynamic data to steady-state shear data. The curve fitting was performed using Polyflow (ANSYS, Inc., Canonsburg, PA, USA) where the Cox-Merz rule [13] was applied to convert the dynamic data to steady-state shear data. The fitted results for each material are given in Table I and Figure 7. Fitted parameters for the Power Law model were based on the angular frequency (or shear rate) interval from 50 rad/s to 280 rad/s (or 50 s⁻¹ to 280 s⁻¹) since this is the average Power Law apparent wall shear rate in region I and III (cf. Figure 1(b)) when the filaments are extruded at 0.5 RPM – 2.5 RPM in our experiments. And 280 rad/s is the maximum angular frequency that MARS 40 can guarantee the accurate data. The fitted parameters of GNF models serve as input for the Bellini, Phan, and enhanced GNF models presented above, such that the prediction of the pressure drop can be calculated and compared to those measured in our filament testing device. The percentage error of the prediction of pressure drop with respect to the measurements are given in Table II-V for four materials.

Table I: Curve-fitting results for each material from MARS 40 data using Polyflow with multiple GNF models

<table>
<thead>
<tr>
<th>Filaments</th>
<th>Power Law ([k (Pa \cdot s^n), n])</th>
<th>Bird-Carreau ([\eta_0 (m^2/s), \lambda(s), n])</th>
<th>Cross ([\eta_0 (m^2/s), \lambda(s), n])</th>
<th>Carreau-Yasuda ([\eta_0 (m^2/s), \lambda(s), n, a])</th>
</tr>
</thead>
<tbody>
<tr>
<td>3DXTECH ABS</td>
<td>[0.736e4, 0.507]</td>
<td>[0.376e4, 0.692, 0.619]</td>
<td>[0.65e4, 0.71, 0.48]</td>
<td>[0.137e5, 0.101e-1, 0.126, 0.209]</td>
</tr>
<tr>
<td>Triotech Plastic ABS</td>
<td>[0.125e5, 0.419]</td>
<td>[0.271e4, 0.153, 0.672]</td>
<td>[0.178e5, 2.050, 0.562]</td>
<td>[0.685e5, 0.618, 0.226, 0.220]</td>
</tr>
<tr>
<td>Triotech Plastic Amphora</td>
<td>[0.552e4, 0.652]</td>
<td>[0.271e4, 0.153, 0.672]</td>
<td>[0.417e4, 0.836e-1, 0.472]</td>
<td>[0.368e4, 0.132, 0.586, 0.618]</td>
</tr>
<tr>
<td>3DXTECH PLA</td>
<td>[0.262e4, 0.710]</td>
<td>[0.1105e4, 0.587e-1, 0.725]</td>
<td>[0.12544, 0.705e-2, 0.620]</td>
<td>[0.129e4, 0.153e-2, 0.148e-5, 0.5173]</td>
</tr>
</tbody>
</table>
Figure 7: Comparison of fitted results for viscosity curves from MARS 40 using Power Law, Bird-Carreau, Cross, and Carreau-Yaduda models for (a), 3DXTECH ABS; (b), TRIPTECH ABS; (c), Amphora; (d), PLA.

Table II: Predictions of pressure drop using each model and the percentage errors with respect to experimental data for ABS (3DXTECH)

<table>
<thead>
<tr>
<th>Filament Feeding Rate (RPM)</th>
<th>Measured Pressure drop ∆P - Pa</th>
<th>Bellini Model ∆P - Pa (% error)</th>
<th>Bird-Carreau ∆P - Pa (% error)</th>
<th>Enhanced GNF Models</th>
<th>Carreau-Yasuda ∆P - Pa (% error)</th>
<th>Phan Model ∆P - Pa (% error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>7.166e5</td>
<td>7.494e5 (4.6)</td>
<td>5.833e5 (-18.6)</td>
<td>5.928e5 (-17.3)</td>
<td>5.977e5 (-16.6)</td>
<td>1.603e6 (47.9)</td>
</tr>
<tr>
<td>1</td>
<td>1.022e6</td>
<td>1.064e6 (4.0)</td>
<td>9.224e5 (-9.8)</td>
<td>9.063e5 (-11.3)</td>
<td>9.030e5 (-11.7)</td>
<td>1.505e6 (47.2)</td>
</tr>
<tr>
<td>1.5</td>
<td>1.276e6</td>
<td>1.302e6 (2.1)</td>
<td>1.196e6 (-6.3)</td>
<td>1.153e6 (-9.6)</td>
<td>1.137e6 (-10.9)</td>
<td>1.843e6 (44.5)</td>
</tr>
<tr>
<td>2</td>
<td>1.498e6</td>
<td>1.506e6 (0.5)</td>
<td>1.436e6 (-4.1)</td>
<td>1.367e6 (-8.7)</td>
<td>1.336e6 (-10.8)</td>
<td>2.131e6 (42.2)</td>
</tr>
<tr>
<td>2.5</td>
<td>1.664e6</td>
<td>1.687e6 (1.4)</td>
<td>1.656e6 (-0.5)</td>
<td>1.560e6 (-6.2)</td>
<td>1.513e6 (-9.1)</td>
<td>2.387e6 (43.5)</td>
</tr>
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Table III: Predictions of pressure drop using each model and the percentage errors with respect to experimental data for ABS (Triptech Plastics)

<table>
<thead>
<tr>
<th>Filament Feeding Rate (RPM)</th>
<th>Measured Pressure drop ∆P - Pa</th>
<th>Bellini Model ∆P - Pa (% error)</th>
<th>Bird-Carreau ∆P - Pa (% error)</th>
<th>Enhanced GNF Models</th>
<th>Carreau-Yasuda ∆P - Pa (% error)</th>
<th>Phan Model ∆P - Pa (% error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>9.658e5</td>
<td>1.007e6 (4.2)</td>
<td>7.835e5 (-18.9)</td>
<td>7.948e5 (-17.7)</td>
<td>8.023e5(-16.9)</td>
<td>1.374e6 (42.3)</td>
</tr>
<tr>
<td>1</td>
<td>1.320e6</td>
<td>1.344e6 (1.8)</td>
<td>1.160e6 (-12.1)</td>
<td>1.140e6 (-13.7)</td>
<td>1.134e6 (-14.1)</td>
<td>1.835e6 (38.9)</td>
</tr>
<tr>
<td>1.5</td>
<td>1.588e6</td>
<td>1.588e6 (0.1)</td>
<td>1.439e6 (-9.3)</td>
<td>1.396e6 (-12.1)</td>
<td>1.376e6 (-13.3)</td>
<td>2.168e6 (36.6)</td>
</tr>
<tr>
<td>2</td>
<td>1.771e6</td>
<td>1.791e6 (1.1)</td>
<td>1.672e6 (-5.6)</td>
<td>1.611e6 (-9.0)</td>
<td>1.577e6 (-11.0)</td>
<td>2.444e6 (38.0)</td>
</tr>
<tr>
<td>2.5</td>
<td>1.917e6</td>
<td>1.967e6 (2.6)</td>
<td>1.877e6 (-2.1)</td>
<td>1.801e6 (-6.1)</td>
<td>1.751e6 (-8.7)</td>
<td>2.684e6 (40.0)</td>
</tr>
</tbody>
</table>
Table IV: Predictions of pressure drop using each model and the percentage errors with respect to experimental data for Amphora

<table>
<thead>
<tr>
<th>Filament Feeding Rate (RPM)</th>
<th>Measured Pressure drop ΔP - Pa</th>
<th>Bellini Model ΔP - Pa (% error)</th>
<th>Enhanced GNF Models</th>
<th>Phan Model ΔP - Pa (% error)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Bird-Carreau ΔP - Pa (% error)</td>
<td>Cross ΔP - Pa (% error)</td>
<td>Carreau-Yasuda ΔP - Pa (% error)</td>
</tr>
<tr>
<td>0.5</td>
<td>9.184e5</td>
<td>8.840e5 (-3.7)</td>
<td>7.775e5 (-15.3)</td>
<td>8.026e5 (-12.6)</td>
</tr>
<tr>
<td>1</td>
<td>1.460e6</td>
<td>1.387e6 (-5.0)</td>
<td>1.268e6 (-13.1)</td>
<td>1.286e6 (-11.9)</td>
</tr>
<tr>
<td>1.5</td>
<td>1.931e6</td>
<td>1.799e6 (-6.8)</td>
<td>1.683e6 (-12.8)</td>
<td>1.676e6 (-13.2)</td>
</tr>
<tr>
<td>2</td>
<td>2.314e6</td>
<td>2.169e6 (-6.3)</td>
<td>2.062e6 (-10.9)</td>
<td>2.020e6 (-12.7)</td>
</tr>
<tr>
<td>2.5</td>
<td>2.690e6</td>
<td>2.510e6 (-6.7)</td>
<td>2.417e6 (-10.2)</td>
<td>2.335e6 (-13.2)</td>
</tr>
</tbody>
</table>

Table V: Predictions of pressure drop using each model and the percentage errors with respect to experimental data for PLA

<table>
<thead>
<tr>
<th>Filament Feeding Rate (RPM)</th>
<th>Measured Pressure drop ΔP - Pa</th>
<th>Bellini Model ΔP - Pa (% error)</th>
<th>Enhanced GNF Models</th>
<th>Phan Model ΔP - Pa (% error)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Bird-Carreau ΔP - Pa (% error)</td>
<td>Cross ΔP - Pa (% error)</td>
<td>Carreau-Yasuda ΔP - Pa (% error)</td>
</tr>
<tr>
<td>0.5</td>
<td>5.412e5</td>
<td>5.130e5 (-5.2)</td>
<td>4.499e5 (-16.9)</td>
<td>4.595e5 (-15.1)</td>
</tr>
<tr>
<td>1</td>
<td>8.723e5</td>
<td>8.375e5 (-4.0)</td>
<td>7.593e5 (-13.0)</td>
<td>7.653e5 (-12.3)</td>
</tr>
<tr>
<td>1.5</td>
<td>1.134e6</td>
<td>1.112e6 (-2.0)</td>
<td>1.025e6 (-9.6)</td>
<td>1.010e6 (-11.0)</td>
</tr>
<tr>
<td>2</td>
<td>1.388e6</td>
<td>1.363e6 (-1.8)</td>
<td>1.272e6 (-8.4)</td>
<td>1.222e6 (-11.9)</td>
</tr>
<tr>
<td>2.5</td>
<td>1.616e6</td>
<td>1.698e6 (-1.2)</td>
<td>1.505e6 (-6.9)</td>
<td>1.413e6 (-12.6)</td>
</tr>
</tbody>
</table>

The enhanced GNF models include more parameters as compared to the Power Law viscosity model. However, all enhanced GNF models predict a lower pressure drop for all materials with higher errors when compared to the Power Law-based Bellini model. Calculations show that values of pressure drop predicted using the enhanced GNF models are sensitive to the Newtonian viscosity (i.e., $\eta_0$). Note that the lowest shear rate (or angular frequency) that can be obtained to guarantee the oscillation frequency sweep test in LVER is near 0.1 s$^{-1}$ (or rad/s) for two brands of ABS and is near 0.5 s$^{-1}$ (or rad/s) for Amphora and PLA using MARS 40. Therefore, viscosity curves in Figure 7 are perhaps insufficient to represent the viscosity under low shear rates for each material, possibly limiting the accuracy for values of $\eta_0$ in our experiments. To obtain more accurate values of $\eta_0$ may require viscosity data over the lower shear rate region. This could be addressed by applying the time temperature superposition (TTSP), however, as discussed in the study conducted by Dealy, et al [14], applying TTSP may be questionable because this method will introduce more errors. Besides, some polymers do not even have a Newtonian plateau, for example in the study conducted by Seppala, et al. [15], no obvious Newtonian plateau was observed for ABS. It is noticed that the errors of using our enhanced GNF models for two brands of ABS in Table II-III are significantly greater at low filament feeding rate (i.e., low shear rate) compared to the errors at high shear rate. This is probably due to $\eta_0$ dominantly affect the prediction of pressure drop at low shear rate.

We employed Phan model followed their assumption that is $m = n$ and $Tr = 5$, first. Calculated results show that, although the Phan model ignores the pressure drop upper cylinder, the results consistently over-predict pressure drop for all materials. Cogswell [7] reported his model may have a pressure drop error for converging flow as high as ±20%, where the elongation viscosity was assumed as three times as shear viscosity. When the Power Law is used as a...
basis for extension flow, then this ratio is simply the Trouton ratio in Phan model. Cogswell and Phan, et al. suggested different ratios to relate the shear viscosity to elongation viscosity, however, both values were based on their assumptions. Phan, et al. numerically adjusted the values of Trouton ratio to fit to their pressure drop measurements instead of directly extracting $Tr$ or $m$ from the elongation viscosity data. Therefore, more reasonable values of both $m$ and $Tr$ measured from experiments are required to further analyze the effect of extension flow on pressure drop if the Power Law model is assumed for elongation thinning behavior of polymers. Boles, et al. [8] also suggested that his model (i.e., $\Delta P_o$) is not guarantee for accurately predicting the pressure drop with an order-of-magnitude estimation. The equation of $\Delta P_o$ in Boles, et al. was developed based only on their specific experiments, where 1.18 and 0.7 are the empirical parameters they suggested. The Phan model is based on both above models where equation of $\Delta P_o$ was directly used without modification. The selecting of values of parameters (e.g., $n$, $k$, $Tr$), especially, $m$ was assumed is equal to $n$, plays a dominant role in prediction of pressure drop. And this could suggest why the errors are significantly high in our study.

For Bellini Model, it can be observed that most of the errors are within 6%, and this favorable result suggests that the efficiency of this model for describing the creeping flow in an FFF nozzle as well as the validity of our device of measuring pressure drop. Though, multiple assumptions made for developing the equation of pressure drop of converging flow in region $I$ in the repeatable results for all four polymers demonstrate the validation of this strategy of predicting pressure drop of polymer melts flow in a geometrically complicated FFF nozzle. It was also noticed that the Bellini model slightly over-predicts pressure drop for two brands of ABS whereas it under predicts pressure drop for Amphora and PLA filament.

It is interesting to note that the Bellini model where the pressure drop is based on pure shear in the polymer melt flow in the upper cylinder was included fits the experimental data very well. Given that each model has various regions that contribute to the overall pressure drop, a relative comparison of the pressure drop in each region is helpful. We consider results at 1.5 RPM as the representative to compare each model, where the fraction of pressure loss in each region was evaluated and given in Table VI-VIII. As expected, the results show that the dominant pressure drop through the nozzle are in the capillary region for all models. However, the pressure drop in the upper cylindrical (i.e., region $I$ in Figure 1(b)) makes a significant contribution to the overall pressure drop, especially, for two ABS filaments using Bellini model and the enhanced GNF models. This suggests that it is necessary to consider the pressure drop in region $I$ into account when evaluating nozzle flow. In addition, the ratio of the length and diameter of the upper cylinder is 8.37 in our setup which is relatively high to support the assumption that the flow in this region is fully-developed such that the pressure drop due to shear cannot be ignored.

*Table VI: Distribution of pressure drop in each region of the FFF nozzle flow for four polymers extruded at 1.5 RPM using Bellini model*

<table>
<thead>
<tr>
<th>Filaments</th>
<th>Temperature (°C)</th>
<th>Pressure Drop in region $I$ (%)</th>
<th>Pressure Drop in region $II$ (%)</th>
<th>Pressure Drop in region $III$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3DXTECH ABS</td>
<td>230</td>
<td>29.2</td>
<td>13.6</td>
<td>57.2</td>
</tr>
<tr>
<td>Triptech Plastic ABS</td>
<td>230</td>
<td>38.1</td>
<td>13.3</td>
<td>48.6</td>
</tr>
<tr>
<td>Triptech Plastic Amphora</td>
<td>230</td>
<td>17.5</td>
<td>13.4</td>
<td>69.1</td>
</tr>
<tr>
<td>3DXTECH PLA</td>
<td>215</td>
<td>14.0</td>
<td>13.1</td>
<td>72.9</td>
</tr>
</tbody>
</table>
Table VII: Distribution of pressure drop in each region of the FFF nozzle flow for four polymers extruded at 1.5 RPM using enhanced GNF models

<table>
<thead>
<tr>
<th>Filaments</th>
<th>T (°C)</th>
<th>Bird-Carreau</th>
<th>Cross</th>
<th>Carreau-Yasuda</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(\Delta P_I) (%)</td>
<td>(\Delta P_{II}) (%)</td>
<td>(\Delta P_{III}) (%)</td>
</tr>
<tr>
<td>3DXTECH ABS</td>
<td>230</td>
<td>18.4</td>
<td>13.7</td>
<td>67.9</td>
</tr>
<tr>
<td>Triptech Plastic ABS</td>
<td>230</td>
<td>28.6</td>
<td>13.8</td>
<td>57.6</td>
</tr>
<tr>
<td>Triptech Plastic Amphor</td>
<td>230</td>
<td>11.4</td>
<td>13.8</td>
<td>74.8</td>
</tr>
<tr>
<td>3DXTECH PLA</td>
<td>215</td>
<td>7.7</td>
<td>13.4</td>
<td>78.9</td>
</tr>
</tbody>
</table>

Table VIII: Distribution of pressure drop in each region of the FFF nozzle flow for four polymers extruded at 1.5 RPM using Phan model

<table>
<thead>
<tr>
<th>Filaments</th>
<th>Temperature (°C)</th>
<th>(\Delta P_{IIe}) (%)</th>
<th>(\Delta P_{II}) (%)</th>
<th>(\Delta P_{III}) (%)</th>
<th>(\Delta P_{III}) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3DXTECH ABS</td>
<td>230</td>
<td>33.7</td>
<td>10.5</td>
<td>15.4</td>
<td>40.4</td>
</tr>
<tr>
<td>Triptech Plastic ABS</td>
<td>230</td>
<td>37.7</td>
<td>11.2</td>
<td>15.5</td>
<td>35.6</td>
</tr>
<tr>
<td>Triptech Plastic Amphor</td>
<td>230</td>
<td>28.4</td>
<td>9.5</td>
<td>15</td>
<td>47.1</td>
</tr>
<tr>
<td>3DXTECH PLA</td>
<td>215</td>
<td>26.7</td>
<td>9.1</td>
<td>14.8</td>
<td>49.4</td>
</tr>
</tbody>
</table>

Summary

This paper presents a relatively simple and low-cost device that can measure the pressure drop of polymers in FFF nozzle flow which agrees well with predictions computed using the Bellini Power-Law-based model. Comparison results between the models that only consider the pure shear effects, i.e., Bellini model and our proposed enhanced GNF models suggest that the Power Law model can describe the shear thinning behavior of polymer melts accurately for the FFF nozzle flow. This is likely due to the processing shear rate of polymers in an FFF nozzle which are mostly in the Power Law shear thinning region. Though the predictions of our enhanced GNF models are less-accurate, most of the errors are within 13% which is still acceptable considering the limitation of fitted parameters of the GNF models. To the best knowledge of authors, it is the first time of employing pressure drop model using more general GNF models including the Bird-Carreau, Cross, and Carreau-Yasuda models to describe the FFF nozzle flow. It is promising to further develop this approach to describe polymer melts flow more generally that is not limited to the small scale of FFF nozzle flow. For example, similar models it may be employed to other polymer processing techniques, e.g., big area additive manufacturing (BAAM) [16]. However, the main challenge of using our enhanced GNF models is to obtain the relatively accurate parameters experimentally (especially \(\eta_0\)).

Computed results using the Phan model shows higher error as compared to other shear-only models. Phan model predictions are based on the values of parameters of \(m\) and \(Tr\) which was assumed according to the shear viscosity curves obtained in this study and the results from Phan, et al. (i.e. \(Tr = 5\)), respectively. Phan, et al. suggested the value of Trouton ration by fitting the
predictions of pressure drop from their model to those measured in their experiments. The reasonable Trouton ratio and $m$ are required from the experiments of extension flow test (i.e., measurements of strain rate dependent extensional viscosity) to further assess Phan model.

**Acknowledgements**

The authors would like to thank Baylor University for financially support this research.

**Bibliography**

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